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Alternative mass-shell renormalization for a minimal supersymmetric Higgs sector

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Abstract. An Aoki–Denner form of the renormalization scheme is suggested for the physical amplitudes in the minimal supersymmetric standard model. With the more explicit wave-function renormalization, the scheme is parameterized by the mass of the physical pseudoscalar (M_A) and the mass of heavy CP-even neutral Higgs (M_H) instead of the conventional M_A, $\tan\beta$ as input. The counterterm of $\tan\beta$ is defined on mass shell perturbatively just within the Higgs sector. The renormalization of gauge-scalar mixings are fixed by proper Ward–Takahashi identities. The effect of the reparameterization is also probed to the radiative correction of the mass of the lightest Higgs.

1 Introduction

In the minimal supersymmetric standard model (MSSM), the masses and couplings of physical bosons are restricted, and at tree level, they can be expressed in terms of merely two free parameters, although supersymmetry (SUSY) is soft broken[1]. These constraints will no doubt receive radiative corrections, particularly from loops of the top quark and its SUSY partners (stops).

If the MSSM is a perturbative theory, as is expected, some of its qualities at low order should be kept somehow up to higher order. For example, the effective potential (EP) approach [2] and the renormalization-group method (RG) [3], with the use of some tree-level relations, give a logarithm correction $\varepsilon \sim 3G_F M_t^4 \sin^2\beta \log(1+m_{\tilde{t}}^2/m_t^2)$[2] for estimating the mass of Higgs bosons with good approximation that can be used in Higgs phenomena[4].

When one considers the momentum dependence of the full set of Green functions in the Higgs sector, one should also be able to make a logarithm correction like ε within the framework of Feynman diagrammatic calculation (FDC). At the same time, FDC is also requested for the simulation of the phenomena on present and future colliders [5], where we have made searching for the (lightest) Higgs boson one of the main goals. For the identification of a Higgs boson found in the Fermi Lab Tevatron, the CERN large hadron collider (LHC), or next linear collider (NLC), information about its mass is very useful. Especially for the study of whether the produced Higgs is a SUSY one, some appropriate SUSY-like [6] simulations (the hard relations among the SUSY parameters are held somehow) for its production cross section and decay width are crucial too.

Thus, instead of abandoning all the tree-level relations in the MSSM, one should investigate which of those simple SUSY constraints can be retained perturbatively in FDC, as well as study how the other variables can be manipulated loop by loop. With this in mind, we noticed [7,8], in which were developed renormalization procedures within the on-mass-shell scheme following [9]. In their representation, the physical mass of the Higgs boson was acquainted with the pole position of the renormalized propagator, and the Higgs phenomena could be predicted systematically [10]. Recently, even two-loop FDC in a similar direction to [9] was developed for the prediction of the parameter $\rho = M_W^2/(M_Z^2\cos^2\theta_w)$ and the mass of Higgs bosons [11]. All these works have already demonstrated the efficacy of FDC.

Thus in this work we seek for an alternative realization motivated by [12] and [13], in which the wave-function renormalization of mass eigenstate is performed more explicitly, and in which the gauge-fixing terms are renormalized simply, to present a practical option for general MSSM perturbative calculations. Such a framework has been established for the two Higgs doublet model (2HDM), for example in [14]; however, the property of the SUSY allows us to obtain more connections in the radiative corrections and allows us to construct an MSSM version. Similar considerations have been adopted in [15] for radiative corrections.

In [15,8,7], as well as in other literature [16], $\tan\beta$, which is not a physical observable, was chosen as a fundamental input of the MSSM; we will refer to this as the β scheme. In these works, the counterterm of β was fixed with an $\overline{\text{MS}}$-like subtraction or fixed in a process-dependent way involving lengthy calculation of triangle Feynman diagrams. Since $\delta\beta$ is frequently used in the MSSM for most FDC, it should be fixed at a definite scale with an ultraviolet (UV)-finite part, whereas for simplicity

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of the loop calculations, the subtraction should be defined through a set of two-point one-particle irreducible (1PI) Green functions but not through the complicated three-point Green functions.

There is a quite similar situation in the SM, in which the gauge symmetry has simplified the counterterm of electric charge δe as a combination of the self-energies of the gauge bosons. Then the renormalization in our scheme is a reasonable attempt, replacing $\tan\beta$ with the mass of heavy CP-even Higgs M_H.

In addition, the mixing of gauge and Higgs bosons will rise notably, because the MSSM is a gauge theory with two scalar doublets. The heavy top quark and scalar tops will in particular contribute a considerable correction to the mixings that will be necessary for the physical process involving the CP-odd neutral Higgs or the charged Higgs bosons. To our knowledge, from the viewpoint of gauge invariance, this subject is less discussed as a part of systematic MSSM renormalization, although various treatments have already been adopted in individual cases from the subtraction of Goldstone propagators. Since the Ward–Takahashi identities (WTI) play an important role for the renormalization of gauge field theory, we tried to generalize the treatment in [12] to this MSSM case for the counterterms of gauge-scalar mixings.

The present paper is organized as follows. In Sect. 2, we introduce the conventions and notations for the MSSM. In Sect. 3, we accomplish the Aoki–Denner form of the renormalization of the MSSM, including the pole mass of the lightest CP-even Higgs boson and the on-mass-shell counterterms of β. In Sect. 4, we give the wave-function renormalization constants of gauge-scalar mixing terms. We briefly discuss the application of these formulas in the last section. Some essential expressions are listed in the appendices.

2 Tree-level structure of the MSSM and notations

In the MSSM, the original $\mathrm{SU}(2)_\mathrm{L}\otimes\mathrm{U}(1)_Y$ gauge-invariant Higgs sector reads,

$$L_{\mathrm{kin}} = \overline{H}_1 D^\dagger_{-\ \mu} D^\mu_- H_1 + \overline{H}_2 D^\dagger_{+\ \mu} D^\mu_+ H_2 \tag{2.1}$$

where $D^\mu_\mp = \partial^\mu \mp (\mathrm{i}/2) g_1 B^\mu - \mathrm{i} g_2 T^a W^{a\ \mu}$. For low-energy phenomena, the Higgs sector of the MSSM has a soft-broken potential with explicit CP conservation,

$$\begin{aligned} V_{\mathrm{soft}} &= m_1^2 \overline{H}_1 H_1 + m_2^2 \overline{H}_2 H_2 - m_3^2(\epsilon_{ab} H_1^a H_2^b + \mathrm{h.c.}) \\ &+ \frac{1}{8} g^2 (\overline{H}_1 H_1 - \overline{H}_2 H_2)^2 - \frac{g_2^2}{2} |\overline{H}_1 H_2|^2, \end{aligned} \tag{2.2}$$

where m_3^2 is defined to be negative and $\epsilon_{12} = -\epsilon_{21} = -1$, $g^2 = g_1^2 + g_2^2$. Now can we clearly count the five parameters of this model, g_1, g_2, m_1^2, m_2^2, and m_3^2 (where a μ has been absorbed into m_1^2 and m_2^2). In the Higgs sector, this model has fewer parameters than 2HDM, so it should be more predictive.

Down to the electroweak scale, scalar fields develop their vacuum expectation value (VEV) $v_1 \neq 0, v_2 \neq 0$ and mix into mass eigenstates. As the components of Φ_1, Φ_2 in [17], the doublets read,

$$\begin{aligned} H_1 &= \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} (v_1 + \phi_1^0 - \mathrm{i}\chi_1^0)/\sqrt{2} \\ -\phi_1^- \end{pmatrix}, \\ H_2 &= \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ (v_2 + \phi_2^0 + \mathrm{i}\chi_2^0)/\sqrt{2} \end{pmatrix}. \end{aligned} \tag{2.3}$$

Neither v_1 nor v_2 is an independent new parameter if we work in the renormalization of the five parameters. They can be induced as functions of the five parameters by means of the minimum point of the potential [18]

$$\frac{\partial V}{\partial v_1} = 0, \quad \frac{\partial V}{\partial v_2} = 0, \quad (\text{all fields} \Rightarrow 0). \tag{2.4}$$

And $v \equiv \sqrt{v_1^2 + v_2^2}$ gives the mass of gauge bosons similarly to what we have in SM.

Furthermore, we choose the mass of pseudoscalar Higgs M_A and the mass of heavier CP-even neutral Higgs M_H as the input parameters (experiment data expected) for the Higgs sector. Then other parameters can be represented by these five independent parameters.

$$\cos^2 2\beta = \frac{M_H^2 (M_A^2 + M_Z^2 - M_H^2)}{M_A^2 M_Z^2} \tag{2.5}$$

$$\begin{aligned} &\tan 2\alpha = \tan 2\beta \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}, \quad m_3^2 = -M_A^2 \sin\beta\cos\beta, \\ &v_1 = v\cos\beta, \quad v_2 = v\sin\beta \end{aligned} \tag{2.6}$$

A superscript 0 has been omitted in each symbol of an electric neutral particle. We shall see later that (2.5) and the first equation of (2.6) can be used to define β and α loop by loop. At tree level, (2.4) explicitly results in

$$\begin{aligned} 0 &= m_3^2 v \sin\beta + v\cos\beta (m_1^2 + M_Z^2 \cos 2\beta/2), \\ 0 &= m_3^2 v \cos\beta + v \sin\beta (m_2^2 - M_Z^2 \cos 2\beta/2). \end{aligned} \tag{2.7}$$

These equations are based on (2.5, 2.6), and mean that m_1^2, m_2^2 can be considered as functions dependent on M_A, M_H, M_Z, M_W, and e. Equations (2.7) will be changed by loop correction, although they lead to simple tree-level mass for Higgs bosons, for example,

$$M_h^2 = \frac{1}{2}[\ M_A^2 + M_Z^2 - \Delta], \quad M_{H^+}^2 = M_A^2 + M_W^2, \tag{2.8}$$

where $\Delta = \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta}$.

The gauge-Goldstone mixing terms must be encountered when the gauge-invariant eigenstates are transformed into the mass eigenstates. For example, for the Z boson there is one term $Z^\mu \partial_\mu G$ with a coefficient $(v_1\cos\beta + v_2\sin\beta)\sqrt{g_1^2 + g_2^2}/2$. In the parameterization taken here, this mixing term becomes

$$\mathcal{L}_{\mathrm{mix}} = -M_Z Z^\mu \partial_\mu G. \tag{2.9}$$

Fortunately, this mixing can be canceled at tree level by the so-called gauge-fixing term

$$\mathcal{L}_{\rm gf} = -\frac{1}{2\alpha_z}(\partial_\mu Z^\mu + \alpha_z M_Z G)^2, \tag{2.10}$$

which is necessary for the quantization of gauge fields. The cancellation mechanics is also valid for the W gauge boson and photon.

Here we have chosen the SM-like gauge-fixing, and in the following calculations, we will adopt the 't Hooft–Feynman gauge, $\alpha_z = \alpha_w = \alpha_\gamma = 1$. Finally, the physical results are gauge-independent.

3 The renormalization procedure in the scheme

A practical procedure of renormalization is expected when perturbative calculations are performed. One mediocre choice is to follow the formulas listed in [14] and to include the virtual SUSY particles into radiative loops, with the statement that the tree-level relations are spoiled. However, another choice in [7,8] has implied that it is not necessary to give up all SUSY relations. As we will see below, our realization will also reduce the number of independent counterterms counting on the SUSY potential (2.2) in the conventions of [12,13]. In principle, the renormalization in [12,13] is equivalent to the one in [9], although the two should not be used in a mixed way, so our manipulation has to be represented from the beginning.

3.1 General framework

Our scheme, as in most cases, defines explicitly the renormalization constants of fields, which are just the mass eigenstates,

$$W_\mu^\pm \Rightarrow Z_W^{1/2} W_\mu^\pm\ , \quad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \Rightarrow \begin{pmatrix} Z_Z^{1/2} & Z_{Z\gamma}^{1/2} \\ Z_{\gamma Z}^{1/2} & Z_\gamma^{1/2} \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix},$$
$$\begin{pmatrix} A \\ G \end{pmatrix} \Rightarrow \begin{pmatrix} Z_A^{1/2} & Z_{AG}^{1/2} \\ Z_{GA}^{1/2} & Z_G^{1/2} \end{pmatrix} \begin{pmatrix} A \\ G \end{pmatrix},$$
$$\begin{pmatrix} H \\ h \end{pmatrix} \Rightarrow \begin{pmatrix} Z_H^{1/2} & Z_{Hh}^{1/2} \\ Z_{hH}^{1/2} & Z_h^{1/2} \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}. \tag{3.1}$$

We intend to omit the subscript R (meaning renormalized) unless the renormalized quantity is not equivalent to the physical one. Here we have not taken the renormalization to the gauge eigenstates used in [7] and [8], such as

$$H_i \Rightarrow Z_{Hi}^{1/2} H_i, \quad B_\mu \Rightarrow Z_B^{1/2} B_\mu, \quad \vec{W}_\mu \Rightarrow Z_W^{1/2}\ \vec{W}_\mu,$$
$$(\xi_{1,2}^{B,W} \Rightarrow 1 + \delta\xi_{1,2}^{B,W}), \tag{3.2}$$

which seem more compact and concise, so we have to seek an alternative way to treat tadpoles and to define $\delta\beta$. As to the input parameters, our scheme prefers the renormalization to the five physical parameters below:

$$\begin{aligned} &M_Z^2 \Rightarrow M_Z^2 + \delta M_Z^2 \quad M_W^2 \Rightarrow M_W^2 + \delta M_W^2 \\ &M_A^2 \Rightarrow M_A^2 + \delta m_A^2 \quad M_H^2 \Rightarrow M_H^2 + \delta m_H^2 \\ &\quad e \Rightarrow e + \delta e \end{aligned} \tag{3.3}$$

where $M_W,\ M_Z$ are the masses of the gauge bosons and e is the electric charge of the electron, and these three are all renormalized with the conventional treatment in electroweak SM [9,12]:

$$\begin{aligned} &\Re e\ \hat{\Sigma}_Z(k^2)|_{k^2=M_Z^2} \\ &\quad = [\Sigma_Z(k^2) - \delta M_Z^2 + \delta Z_Z(k^2 - M_Z^2)]_{k^2=M_Z^2} = 0 \\ &\Re e\ \hat{\Sigma}_W(k^2)|_{k^2=M_W^2} \\ &\quad = [\Sigma_W(k^2) - \delta M_W^2 + \delta Z_W(k^2 - M_W^2)]_{k^2=M_W^2} = 0 \\ &\hat{\Gamma}^\mu_{\gamma e\bar{e}}(k^2 = 0, \not{p} = \not{q} = m_e) = \mathrm{i}e\gamma^\mu \end{aligned} \tag{3.4}$$

The Green function with a hat is the renormalized one. Regarded as the physical mass of the pseudoscalar (heavy CP-even neutral) Higgs boson, the M_A (M_H) in (3.3) fixes its corresponding counterterm in the same way as the first one in (3.4); this will be demonstrated later.

In addition to the wave-function renormalization of the charged Higgs, the renormalization of physical Higgs masses,

$$M_h^2 \Rightarrow M_h^2 + \delta M_h^2 \quad M_{H^+}^2 \Rightarrow M_{H^+}^2 + \delta M_{H^+}^2, \tag{3.5}$$

makes the renormalization of 2HDM (including the MSSM) complete. In the conventional treatment [21,14], (3.5) fixed the counterterms of Higgs masses, and gave no information on the value of these masses. However, in the MSSM we can give an alternative interpretation to (3.5) from which we can obtain the magnitude of Higgs masses.

3.2 Constraints on mass counterterms

In the MSSM, the mass relations between gauge and Higgs bosons are connected by (2.7), (2.5), and (2.6). Since the renormalization should not increase the number of independent (free) parameters, we have to reproduce the masses of bosons through the breaking of gauge symmetry to investigate how those connections can be regulated by the loop corrections. In other words, the relation between different counterterms should be determined by these constraints. Then the rescaling of the scalar's VEV (i.e., the renormalization of those constraints) is of prime importance. For convenience, we build a generalized form of (2.7)

$$\begin{aligned} \mathcal{T}_h = \frac{v}{8}\{&[8m_3^2\cos\beta + \sin\beta(8m_2^2 - g^2v^2\cos 2\beta)]\cos\alpha \\ &-[8m_3^2\sin\beta + \cos\beta(8m_1^2 + g^2v^2\cos 2\beta)]\sin\alpha\} \\ \mathcal{T}_H = \frac{v}{8}\{&[8m_3^2\sin\beta + \cos\beta(8m_1^2 + g^2v^2\cos 2\beta)]\cos\alpha \\ &+[8m_3^2\cos\beta + \sin\beta(8m_2^2 - g^2v^2\cos 2\beta)]\sin\alpha\}. \end{aligned} \tag{3.6}$$

The v_1, v_2 should generate properly the masses of gauge bosons and fall into the last two of (2.6) order by order. To generate the masses of Higgs bosons, it is convenient to employ the original coefficient matrices in the bilinear terms of scalar fields. The matrices (compressed by a factor of 1/2) have been rotated off gauge basis but their being identified as the physical masses are not necessary. for the identification of the physical masses. The most important one is the mass form of the pseudoscalar A,

$$\mathcal{M}_{AA} = 2[\,(-4m_1^2 + 4m_2^2 - g^2v^2\cos 2\beta)\cos 2\vartheta \\ +4(m_1^2 + m_2^2 - 2m_3^2 \sin 2\vartheta)\,]/16, \qquad (3.7)$$

which is simply an equation to solve m_1^2, m_2^2, and m_3^2, coupled with (3.6). It is easy to check from (A.2) in the appendices that m_1^2, m_2^2, m_3^2 return to their tree-level form (2.7), (2.5), (2.6) only if $\vartheta \Rightarrow \beta, \mathcal{T}_h \Rightarrow 0, \mathcal{T}_H \Rightarrow 0$. Then the mass matrices (quadratic form) of the Higgs sector can be recast as functions of $e, M_W, M_Z, M_A, M_H^2(or\ \tan\beta)$.

These principal parameters (e, M_W, M_Z, M_A, M_H) can run from their bare values to the corresponding renormalized (physical) one, as described in (3.3). When such a replacement is performed, a natural renormalization condition shows up,

$$\mathcal{T}_h^R \equiv \mathcal{T}_h(e_R, M_{WR}, M_{ZR}, M_{AR}, M_{HR}) = 0, \\ \mathcal{T}_H^R \equiv \mathcal{T}_H(e_R, M_{WR}, M_{ZR}, M_{AR}, M_{HR}) = 0. \quad (3.8)$$

This indicates nothing more than that the physical rescaling of VEV eliminates the linear terms of physical Higgs fields, so that each renormalized (one-point) Green function has a tree-level form in renormalized parameters, and so that v_{iR} is just the position where the Higgs potential reaches its minimum. The reasonable relations

$$v_1\delta v_1 + v_2\delta v_2 = v\delta v, \quad \frac{v_2}{v_1}\left(\frac{\delta v_2}{v_2} - \frac{\delta v_1}{v_1}\right) = \sec^2\beta\,\delta\beta \quad (3.9)$$

rather than $\delta v_2 = \delta v_1 = \delta v = 0$ have been used in this scheme. To demonstrate how $\delta\beta$ is traded for δM_H^2, we write the other two transformations,

$$\mathcal{M}_{AA}(e, M_W, M_Z, M_A, M_H) \Rightarrow M_A^2 + \delta M_A^2\,, \\ \mathcal{M}_{HH}(e, M_W, M_Z, M_A, M_H) \\ \equiv [(4m_1^2 + 4m_2^2 + g^2v^2) + 2(2m_1^2 - 2m_2^2 + g^2v^2\cos 2\beta) \\ \times \cos 2\alpha + 2(4m_3^2 - g^2 \sin\beta\cos\beta)\sin 2\alpha]/8 \\ = \frac{1}{2}\left[M_A^2 + M_Z^2 + \Delta\right] + \mathcal{T}_{HH} \Rightarrow \mathcal{M}_{HH}^R + \{\delta\mathcal{M}_{HH}\} \\ \equiv \left[M_H^2 + \mathcal{T}_{HH}^R\right] + \left\{\left[\left(\frac{1}{2} + \frac{\partial\Delta}{\partial M_A^2}\right)\delta M_A^2 \right.\right. \\ \left.\left. + \left(\frac{1}{2} + \frac{\partial\Delta}{\partial M_Z^2}\right)\delta M_Z^2 + \frac{\partial\Delta}{\partial\beta}\delta\beta\right] + T_{HH}\right\} \\ = \left[M_H^2 + \mathcal{T}_{HH}^R\right] + \left\{\left[\delta M_H^2\right] + T_{HH}\right\}, \qquad (3.10)$$

where $\mathcal{T}_{HH}$ is a linear combination of $\mathcal{T}_h, \mathcal{T}_H$ and $\mathcal{T}_{HH}^R = 0$. With T_h and T_H, which are the counterterms of neutral CP-even Higgs tadpoles, equations (3.9,3.7) as well as equations

$$\mathcal{T}_h(e, M_W, M_Z, M_A, M_H) \Rightarrow \mathcal{T}_h^R + T_h = 0 + T_h, \\ \mathcal{T}_H(e, M_W, M_Z, M_A, M_H) \Rightarrow \mathcal{T}_H^R + T_H = 0 + T_H, \quad (3.11)$$

can induce

$$\mathcal{M}_{AG}(e, M_W, M_Z, M_A, M_H) \\ \Rightarrow [\mathcal{M}_{AG}^R] + \delta\mathcal{M}_{AG} \\ = \left[\frac{M_A^2\,(v_2\cos\vartheta - v_1\sin\vartheta)}{v_1\cos\vartheta + v_2\sin\vartheta} + \mathcal{T}_{AG}^R\right] + T_{AG} \\ \mathcal{M}_{hh}(e, M_W, M_Z, M_A, M_H) \\ \Rightarrow M_{hR}^2 + \mathcal{T}_{hh}^R + [\delta M_h^2 + T_{hh}], \qquad (3.12)$$

where M_{hR}^2 obeys (2.8), and $\delta\mathcal{M}_{hh}$ is just its variation,

$$\delta\mathcal{M}_{hh} \equiv \delta M_h^2 + T_{hh} = T_{hh} + \left(\frac{1}{2} - \frac{\partial\Delta}{\partial M_A^2}\right)\delta M_A^2 \\ + \left(\frac{1}{2} - \frac{\partial\Delta}{\partial M_Z^2}\right)\delta M_Z^2 - \frac{\partial\Delta}{\partial\beta}\delta\beta. \qquad (3.13)$$

Here rotational matrices for scalar fields have been defined with different angles, as shown in (A.1). An easy algebra shows that $\delta\vartheta$ (or $\delta\vartheta^+$) can be canceled automatically and neatly by $\delta\beta$, only if $\beta = \vartheta = \vartheta^+$ is set in the coefficients of these counterterms. This pleasing result implies that in this scheme, the angles for Higgs coupling to other particles can be kept as only one angle, i.e., β. In addition, single $\delta\beta$ is sufficient and consistent for any one-loop calculations.

3.3 M_{hP}^2 as the pole mass squared

Combined with the field renormalization in (3.1), the mass counterterms mentioned above can be fixed by the so-called on-mass-shell renormalization conditions in the Higgs sector as follows.

(i) The tadpoles:

$$0 = \mathcal{T}_h^R + T_h + t^h, \quad 0 = \mathcal{T}_H^R + T_H + t^H. \quad (3.14)$$

(ii) The heavy neutral CP-even Higgs:

$$\frac{\mathrm{d}}{\mathrm{d}q^2}\Sigma_{HH}(q^2)|_{q^2=M_H^2} + Z_{HH} + Z_{Hh} = 1 \\ \Re e\ \hat{\Sigma}_H(q^2)|_{q^2=M_H^2} \\ = \left[\Sigma_{HH}(q^2) + (Z_H + Z_{hH})q^2\right]|_{q^2=M_H^2} \\ -2Z_H^{1/2}Z_{Hh}^{1/2}\delta\mathcal{M}_{Hh} - Z_H(\mathcal{M}_{HH} + \delta\mathcal{M}_{HH}) \\ -Z_{hH}(\mathcal{M}_{hh} + \delta\mathcal{M}_{hh}) = 0 \\ \Re e\ \hat{\Sigma}_{Hh}(q^2)|_{q^2=M_H^2} \\ = \left[\Sigma_{Hh}(q^2) + \left(Z_{Hh}^{1/2}Z_H^{1/2} + Z_h^{1/2}Z_{hH}^{1/2}\right)q^2\right]|_{q^2=M_H^2} \\ -Z_{hH}^{1/2}Z_h^{1/2}(\mathcal{M}_{hh} + \delta\mathcal{M}_{hh}) \\ -Z_{Hh}^{1/2}Z_H^{1/2}(\mathcal{M}_{HH} + \delta\mathcal{M}_{HH}) \\ -\left(Z_h^{1/2}Z_H^{1/2} + Z_{Hh}^{1/2}Z_{hH}^{1/2}\right)\delta\mathcal{M}_{Hh} = 0. \qquad (3.15)$$

(iii)The light neutral CP-even Higgs:

$$\frac{\mathrm{d}}{\mathrm{d}q^2}\ \Sigma_{hh}(q^2)+Z_{hh}+Z_{hH}=1$$
$$\Re e\ \hat{\Sigma}_{hh}(q^2)$$
$$=\left[\Sigma_h(q^2)+(Z_h+Z_{Hh})q^2\right]-Z_h(\mathcal{M}_{hh}+\delta\mathcal{M}_{hh})$$
$$-Z_{Hh}(\mathcal{M}_{HH}+\delta\mathcal{M}_{HH})-2Z_h^{1/2}Z_{Hh}^{1/2}\delta\mathcal{M}_{Hh}=0$$
$$\Re e\ \hat{\Sigma}_{hH}(q^2)$$
$$=\left[\Sigma_{Hh}(q^2)+\left(Z_h^{1/2}Z_{hH}^{1/2}+Z_{Hh}^{1/2}Z_H^{1/2}\right)q^2\right]$$
$$-Z_{hH}^{1/2}Z_h^{1/2}\left(\mathcal{M}_{hh}+\delta\mathcal{M}_{hh}\right)$$
$$-Z_{Hh}^{1/2}Z_H^{1/2}(\mathcal{M}_{HH}+\delta\mathcal{M}_{HH})$$
$$-\left(Z_H^{1/2}Z_h^{1/2}+Z_{Hh}^{1/2}Z_{hH}^{1/2}\right)\delta\mathcal{M}_{Hh}=0. \qquad (3.16)$$

The subtraction for the neutral CP-odd Higgs A has the same form as (3.15) when a field substitution $\{H,h\}\Rightarrow\{G,A\}$ ($M_G^2=0$) is made. The symbols t^h, t^H, Σ_{HH}, Σ_{hh}, Σ_{hH}, Σ_{AA} and Σ_{GA} denote the 1PI Green functions (loop momentum integrals with UV divergence). For the sector that concerns the selected input parameters, it is easy to solve:

$$T_h=-t^h,\quad T_H=-t^H$$
$$\delta M_H^2=\Sigma_{HH}(M_H^2)-T_{HH},\quad \delta Z_H=-\frac{\mathrm{d}}{\mathrm{d}q^2}\Sigma_{HH}(M_H^2)$$
$$\delta Z_{hH}=\frac{2[T_{hH}-\Sigma_{hH}(M_H^2)]}{M_H^2-M_{hR}^2}$$
$$\delta M_A^2=\Sigma_{AA}(M_A^2),\quad \delta Z_A=-\frac{\mathrm{d}}{\mathrm{d}q^2}\Sigma_{AA}(M_A^2),$$
$$\delta Z_G=-\frac{\mathrm{d}}{\mathrm{d}q^2}\Sigma_{GG}(0)$$
$$\delta Z_{GA}=\frac{2[T_{GA}-\Sigma_{GA}(M_A^2)]}{M_A^2}, \qquad (3.17)$$

in which (3.10) and (3.12) have been used.

It is notable that the variable in Σ_{HH} is the physical mass M_H^2 in (3.15), while it is also M_{HR}^2. On the other hand, the variable of Σ_{hh} in (3.16) is not M_{hR}^2 but the q^2 to solve. The physical (pole) mass of the light Higgs can be solved as a function of M_A^2, M_H^2, M_Z^2, e. The Taylor expansion may simplify the analysis and help us to realize this point.

$$q^2=M_{hR}^2+\frac{1}{2}(q^2-M_{hR}^2)^2\frac{\mathrm{d}^2}{\mathrm{d}^2q^2}\Sigma_{hh}(q^2)|_{q^2=M_{hR}^2}$$
$$+\mathcal{O}\left(\left(q^2-M_{hR}^2\right)^3\right)+\frac{1}{2}\left[\delta M_Z^2+\delta M_A^2-\delta\Delta\right]$$
$$-\Sigma_{hh}(M_{hR}^2)+T_{hh} \qquad (3.18)$$

The choice $q^2=M_{HR}^2$ in (3.15) makes (3.16) independent of $\delta\Delta$ (i.e., $\delta\beta$) when these two equations are combined together:

$$q^2=M_Z^2+M_A^2-M_H^2+\frac{1}{2}(q^2-M_{hR}^2)^2\frac{\mathrm{d}^2}{\mathrm{d}^2q^2}$$
$$\times\Sigma_{hh}(q^2)|_{q^2=M_{hR}^2}+\mathcal{O}\left(\left(q^2-M_{hR}^2\right)^3\right)$$
$$-\left[\Sigma_{hh}\left(M_{hR}^2\right)+\Sigma_{HH}\left(M_H^2\right)\right]+\left[\delta M_Z^2+\delta M_A^2\right]$$
$$+\left[T_{hh}+T_{HH}\right] \qquad (3.19)$$

The $\mathrm{d}^2/(\mathrm{d}^2q^2)\Sigma_{hh}(q^2)|_{q^2=M_{hR}^2}$ term is UV finite unless the order of divergence in the self-energies of the scalars are higher than quadratic. Had not the SUSY been broken, the remaining $\Sigma_{hh}(M_h^2), \Sigma_{HH}(M_H^2), \Sigma_{AA}(M_A^2), \Sigma_{ZZ}^T(M_Z^2)$ and T_{HH}, T_{hh} would also be convergent in a superrenormalizable theory. Although the breaking of SUSY causes those tadpoles and self-energies to diverge, we can put forward the question of whether the cancellation will remain accommodated.

The possibility for the last line in (3.19) to be UV finite was hinted at in the Appendix E.7 of the second report in [1] and also was indirectly verified analytically in [19]. Our combination of self-energies for Z^μ, A, H, and h bosons in (3.19) were employed as the "renormalization of the neutral Higgs boson mass sum rule" in [15,20].

We have examined it with the top quark and its squarks in one-loop corrections. We can manifest the UV divergence in these $4\times7+2\times3=36$ diagrams to cancel neatly.

3.4 M_H input scheme and the counterterm of β

One may have noticed our using (2.5) for (3.19). This means we have defined β as an induced variable through (2.5) perturbatively. This definition requires the validity of a relation in the MSSM at tree level,

$$M_H^2-M_Z^2<M_A^2<M_H^2. \qquad (3.20)$$

It is known that there is no experiment so far that rules out such an inequality, although LEP does not prefer an SM Higgs to be lighter than gauge boson Z [22]. Here, we think it can be a possible testing point for the MSSM.

There is also no theoretical evaluation that opposes (3.20). In fact, the M_{HP}^2 predicted in the β scheme is destined never to be in conflict with expression (3.20). The EP [2] performed few numerical evaluations for M_{HP}^2. One typical numerical evolution in the RG approach can be found in the first paper of [3], and our (3.20) overlaps most of the permitted region in its Fig. 4b, when their $R=v_2/v_1$ is larger than 1. Even those recent works that include stop mixing and RG improvement, for example [24], cannot definitely contradict the spectrum (3.20). In these terms [25], our spectrum translates to $M_{hP}^2-M_Z^2<\varepsilon<M_{hP}^2$, which is a natural parameter space. In the previous FDC evaluations, the figures in [7,23] confirmed the same spectra when $\tan\beta>1$.

Those arguments in the β scheme for the smallness of $\delta\beta$ are just what we need to support the loop corrections that will perturb (2.5) mildly enough, so we start utilizing M_H^2 directly instead of using β as a fundamental input parameter: $M_H^2\equiv M_{H\mathrm{Phys}}^2=M_{HR}^2$. The measurement of M_H^2 may not occur as early as we expect, but its physical definition is always more unequivocal than the physical definition of β itself. (Another attempt to reparameterize was made in [7]. There, β was replaced by the mass of the lightest Higgs boson M_h^2, which may be measured first.)

The counterterm of M_H^2 then leads to the one of β through (2.5),

$$\delta\beta = \epsilon\frac{\delta Z\ (H-A)HA}{M_{ZHA}} + \frac{\delta H\ (A+Z-2H)AZ + \delta A\ (H-Z)HZ}{M_{ZHA}}, \quad (3.21)$$

where $\epsilon \equiv \mathrm{Sign}(v_2 - v_1)$. A, H, and Z denote M_A^2, M_H^2, and M_Z^2, respectively; δA means δM_A^2, and so on, and

$$M_{ZHA} = 4AZ\sqrt{H(H-Z)(H-A)(A+Z-H)}. \quad (3.22)$$

It is analogous with the renormalization of the Weinberg angle in SM. There the radiative corrections do not frighten one away from defining the counterterm of $\delta\theta_W$ by $\cos^2\theta_W = M_W^2/M_Z^2$ even before the discovery of gauge bosons W and Z.

The renormalization constants of other mixings are obtained as soon as M_{hP}^2 is reached:

$$\delta Z_{Hh} = 2\left[\frac{\Sigma_{Hh}(M_{hP}^2)}{M_{HP}^2 - M_{hP}^2} + \frac{T_{Hh}}{(M_{HP}^2 - M_{hR}^2)} + \frac{(M_{hP}^2 - M_{hR}^2)\Sigma_{Hh}(M_{HP}^2)}{(M_{HP}^2 - M_{hR}^2)(M_{HP}^2 - M_{hP}^2)}\right]$$
$$\delta Z_h = -\frac{\mathrm{d}}{\mathrm{d}q^2}\Sigma_{hh}(q^2)|_{q^2=M_{hP}^2}; \quad (3.23)$$

they are useful for practical manipulation. Similar treatment can be applied to the charged Higgs as listed in the appendices. The frequently used rotation angle α of CP-even Higgs is defined in the first of (2.6), and its counterterm is

$$\delta\alpha = \frac{1}{2}\csc^2 2\alpha\left[2\csc^2 2\beta\frac{M_Z^2 + M_A^2}{M_A^2 - M_Z^2}\delta\beta + 2\tan 2\beta\frac{M_Z^2\delta M_Z^2 - M_A^2\delta M_A^2}{(M_A^2 - M_Z^2)^2}\right] \quad (3.24)$$

Now, the radiatively corrected tree-level relations are only (2.8). Even these equations will remain valid unless we insist that M_{hR}^2 (M_{H+R}^2) is a physical mass of (charged) Higgs. The perturbative MSSM permits a chance to calculate the dependence of M_{hP}^2, M_{H+P}^2, β, and α on e, M_Z^2, M_W^2, M_A^2, and M_H^2, which will be given as input parameters from experiments.

A numerical investigation is given in Sect. 5, where we will show that the reparameterization $\beta \Rightarrow M_H^2$ has not caused any considerable numerical distinction from other schemes. Now we turn to another important aspect of MSSM renormalization, the consequence of gauge invariance.

4 Renormalization of gauge-Higgs mixing from WTI

The appropriate renormalization of the gauge-fixing term (and consequently of gauge-Higgs mixing) should be consistent with the renormalization of VEV (and consequently of β); we study this carefully below. Following [27] and [12], we change nothing else but attach a subscription R to the fields and parameters in (2.10). Then, in our scheme, the renormalization keeps the form of (2.10) unchanged. The main reason is that our renormalization procedure on the classical Lagrangian has already been built to cancel all the UV divergence in the proper vertices. It is convenient to examine this point by an auxiliary generating functional action [12]

$$\bar{\Gamma}[F,K] = -\mathrm{i}\log\left\{\int[\mathcal{D}F(x)]\,\mathrm{Exp}\left[\mathrm{i}\int\mathrm{d}x\mathcal{L}(x)_{\mathrm{eff}}\right] + J(x)F(x) + K(x)\delta F(x)\right\} + \mathrm{i}\log\left\{\int[\mathcal{D}F(x)]\,\mathrm{Exp}\left[\mathrm{i}\int\mathrm{d}z\mathcal{L}(z)_{\mathrm{gf}}\right]\right\}, \quad (4.1)$$

where $\int[\mathcal{D}F(x)]$ denotes the functional integrating of all the fields $F(x)$, such as vector, scalar, and ghost fields. $\delta F(x)$ is the variation of $F(x)$ under the so-called Becchi–Rouet–Stora–Tyutin (BRST) transformation [26], and $K(x)$ is the corresponding external source. The contribution from the BRST transformation term is added to

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{cl} + \mathcal{L}_{\mathrm{gf}} + \mathcal{L}_{FP} \quad (4.2)$$

Since there is only quadratic field production in the gauge-fixing term, the contribution from $\mathcal{L}_{\mathrm{gf}}$ in $\bar{\Gamma}[F,K]$ will be made to the physical proper vertices merely within the inner propagator of loops. So the deduction of $\mathcal{L}_{\mathrm{gf}}$ in (4.1) means that no renormalization substitution is needed for $\mathcal{L}_{\mathrm{gf}}$ within $\mathcal{L}(x)_{\mathrm{eff}}$. Further expounding necessitates the important WTI (strictly speaking, it is the Slavnov–Taylor identity), which is held in both 2HDM and the MSSM,

$$\int\mathrm{d}^4x\frac{\delta\bar{\Gamma}}{\delta Z_\nu(x)}\frac{\delta\bar{\Gamma}}{\delta K_Z^\nu(x)} + \frac{\delta\bar{\Gamma}}{\delta A_\nu(x)}\frac{\delta\bar{\Gamma}}{\delta K_\gamma^\nu(x)} + \frac{\delta\bar{\Gamma}}{\delta G(x)}\frac{\delta\bar{\Gamma}}{\delta K_G(x)} + \frac{\delta\bar{\Gamma}}{\delta A(x)}\frac{\delta\bar{\Gamma}}{\delta K_A(x)} + \frac{\delta\bar{\Gamma}}{\delta C^Z(x)}\frac{\delta\bar{\Gamma}}{\delta K_{C^Z}(x)} + \frac{\delta\bar{\Gamma}}{\delta C^\gamma(x)}\frac{\delta\bar{\Gamma}}{\delta K_{C^\gamma}(x)} = 0, \quad (4.3)$$

where the symbolic $F(x)$ has been embodied as the neutral vector boson Z_μ (photon A_μ or γ) and its corresponding ghost C^Z (C^γ), the neutral unphysical Goldstone G, and the pseudoscalar A. The $K_i(x)$ is the BRST source coupled to corresponding fields respectively. When the functional differentiates $\delta^2/(\delta Z_\mu(y_1)\delta C^Z(y_2))$, $\delta^2/(\delta G(y_1)\delta C^Z(y_2))$, $\delta^2/(\delta A(y_1)\delta C^Z(y_2))$, $\delta^2/(\delta A_\mu(y_1)\delta C^Z(y_2))$, $\delta^2/(\delta A_\mu(y_1)\delta C^\gamma(y_2))$, and $\delta^2/(\delta A_\mu(y_1)\delta C^Z(y_2))$ are performed upon (4.3), a set of WTI such as (C.1) are obtained.

There, $G^a_{i,j}$ denotes an *auxiliary* two-point 1PI vertex in momentum space, and $\tilde{\Gamma}[C^i, K_j]$ denotes the Fourier transformation of $(\delta^2\bar{\Gamma})/(\delta C^i \delta K_j)$. The latter can be calculated from

$$\begin{aligned}
\delta G &= \frac{gv}{2}C^Z - \frac{g_1}{2}[C^+G^- + C^-G^+] \\
&\quad + \frac{g}{2}C^Z[H\cos(\alpha-\beta) - h\sin(\alpha-\beta)] \\
\delta A &= \frac{g}{2}C^Z[h\cos(\alpha-\beta) + H\sin(\alpha-\beta)] \\
&\quad - \frac{g_1}{2}[C^+H^- + C^-H^+] \\
\delta Z_\mu &= -\frac{\mathrm{i}g_1^2}{g}(W^+_\mu C^- - W^-_\mu C^+) + \partial_\mu C^Z \\
\delta A_\mu &= -\frac{\mathrm{i}g_1 g_2}{g}(W^+_\mu C^- - W^-_\mu C^+) + \partial_\mu C^A. \qquad (4.4)
\end{aligned}$$

For example, we have

$$\tilde{\Gamma}[C^Z, K^\nu_Z] = k_\nu J(k^2), \quad \tilde{\Gamma}[C^Z, K_G] = -\mathrm{i}M_Z I(k^2) \quad (4.5)$$

and so on. It is lucky to find that for most of these physical vertices $G^a_{i,j}$, their coefficient functions usually vanish at tree level. Those unphysical vertices $\tilde{\Gamma}[C^i, K_j]$ can be eliminated away as they had been treated in [12] even in higher-order perturbative manipulation. Furthermore, when the corrections of $G^a_{i,j}$ are considered merely to one-loop level, $\tilde{\Gamma}[C^i, K_j]$ can be kept at lower order; then the tree-level form $J(k^2) = 1,\ I(k^2) = 1$ is sufficient for these equations,

$$\begin{aligned}
&G^{\mu\nu}_{ZZ}k_\nu + G^\mu_{ZG}(-\mathrm{i}M_Z) = 0, \quad G^\nu_{GZ}k_\nu + G_{GG}(-\mathrm{i}M_Z) = 0 \\
&G^\nu_{AZ}k_\nu + G_{AG}(-\mathrm{i}M_Z) = 0, \quad G^{\mu\nu}_{\gamma Z}k_\nu + G^\mu_{\gamma G}(-\mathrm{i}M_Z) = 0 \\
&G^{\mu\nu}_{\gamma Z}k_\nu = 0, \quad G^\nu_{A\gamma}k_\nu = 0. \qquad (4.6)
\end{aligned}$$

Except for the third and the sixth, these equations are recognized just as the ones known in the SM. For example, [9]gave similar expressions by means of the generating functional of the full Green function.

With the definition of the renormalization constants $Z_{ZG}, \delta Z_{AZ}, \delta Z_{\gamma G}$, and $\delta Z_{A\gamma}$ from

$$\begin{aligned}
\hat{G^\mu}_{ZG} &= Z_{ZG}(-\mathrm{i}M_Z k^\mu) + G^\mu_{ZG}, \\
\hat{G^\mu}_{GZ} &= Z_{ZG}(\mathrm{i}M_Z k^\mu) + G^\mu_{GZ} \\
\hat{G^\mu}_{AZ} &= \delta Z_{AZ}(\mathrm{i}M_Z k^\mu) + G^\mu_{AZ}, \\
\hat{G^\mu}_{\gamma G} &= \delta Z_{\gamma G}(-\mathrm{i}M_Z k^\mu) + G^\mu_{\gamma G} \\
\hat{G^\mu}_{A\gamma} &= \delta Z_{A\gamma}(\mathrm{i}M_Z k^\mu) + G^\mu_{A\gamma}, \qquad (4.7)
\end{aligned}$$

the equations in (4.6) can also constrain the renormalized vertices,

$$\begin{aligned}
&k_\nu\left[(M_Z^2 + \delta M_Z^2)Z_Z\frac{k^\mu k^\nu}{k^2} + G^{\mu\nu}_{ZZ}\right] \\
&\quad + (-\mathrm{i}M_Z)\left[Z_{ZG}(-\mathrm{i}M_Z k^\nu) + G^\mu_{ZG}\right] = 0 \\
&k_\nu\left[Z_{ZG}(\mathrm{i}M_Z k^\nu) + G^\mu_{GZ}\right] + (-\mathrm{i}M_Z)[Z_G k^2 + G_{GG}] = 0 \\
&k_\nu\left[\delta Z_{ZA}(\mathrm{i}M_Z k^\nu) + G^\mu_{AZ}\right] \\
&\quad + (-\mathrm{i}M_Z)[Z^{1/2}_{GA}M_A^2 + G_{AG}] = 0 \\
&k_\nu[(M_Z^2 + \delta M_Z^2)Z_Z^{1/2}Z_{Z\gamma}^{1/2}\frac{k^\mu k^\nu}{k^2} \\
&\quad + G^{\mu\nu}_{Z\gamma}] + (-\mathrm{i}M_Z)[\delta Z_{\gamma G}(-\mathrm{i}M_Z k^\nu) + G^\mu_{\gamma G}] = 0 \\
&k_\nu[(M_Z^2 + \delta M_Z^2)Z_{Z\gamma}\frac{k^\mu k^\nu}{k^2} + G^{\mu\nu}_{\gamma Z}] = 0, \\
&k_\nu\left[\delta Z_{A\gamma}(\mathrm{i}M_Z k^\nu) + G^\mu_{A\gamma}\right] = 0 \qquad (4.8)
\end{aligned}$$

When we contract the first one in (4.8) with a k_μ and add it to the second one which is produced with an $\mathrm{i}M_Z$, the Z_{ZG} will be fixed naturally. The last two are the trivial constraints, which repeat the fact that the propagator of massless photon is transverse and that A^μ–A mixing is UV-convergent at one-loop level.

$$\begin{aligned}
\delta Z_{ZG} &= \frac{1}{2}\delta Z_Z + \frac{1}{2}\delta Z_G + \frac{\delta M_Z^2}{2M_Z^2} \\
\delta Z_{AZ} &= \frac{1}{2}\delta Z_{GA} \quad \delta Z_{\gamma G} = \frac{1}{2}\delta Z_{Z\gamma} \quad \delta Z_{\gamma A} = 0 \qquad (4.9)
\end{aligned}$$

We know that it is the proper vertices that go into physical calculations and that $\bar{\Gamma}$ in (4.1) is just the generating functional of proper vertices if its gauge-fixing term is restored. When we return the gauge-fixing term to $\bar{\Gamma}$, only the unit term in the original gauge-scalar mixing is canceled, since the gauge-fixing terms are kept unchanged in this scheme. The true renormalized proper vertex is given by our scheme,

$$\begin{aligned}
\hat{\Gamma^\mu}_{ZG} &= \delta Z_{ZG}(-\mathrm{i}M_Z k^\nu) + \hat{G^\mu}_{ZG} \\
\hat{\Gamma^\mu}_{AZ} &= \hat{G^\mu}_{AZ}, \quad \hat{\Gamma^\mu}_{\gamma G} = \hat{G^\mu}_{\gamma G}. \qquad (4.10)
\end{aligned}$$

It is worthwhile to notice that the $\delta\mathcal{M}_{AG}$ (the tadpole in (3.12)) must be included in G_{AG}. Otherwise, the UV divergence in the Z_μ–G transition can not be canceled neatly; this is formally stated by the third of (4.6). Since these renormalization constants of mixing ought to be inserted into the amplitude of some physical processes, the tadpoles should not be dropped away naively; thus they support our definition of $\delta\beta$ for a UV-finite S Matrix. In such a way, when the relations of proper vertices are constructed appropriately, (4.6) guarantees that such a perturbative definition can make the renormalized Green functions in the left side of (4.7) be UV-convergent.

For comparison, we perform the renormalization replacement up to the linear order of δ listed in (3.1) but leave $\mathcal{L}_{\mathrm{gf}}$ unchanged. After having collected the dimensionless coefficients of the gauge-scalar fixings $Z_\mu\partial^\mu G$, $Z_\mu\partial^\mu A$, $A_\mu\partial^\mu G$, and $A_\mu\partial^\mu A$, respectively, we find run into (4.9) again.

$$\begin{aligned}
\mathcal{L}_{\mathrm{mix}} &= -M_Z Z^\mu \partial_\mu G \\
&\Rightarrow -(M_Z + \delta M_Z)\left[Z_Z^{1/2}Z^\mu + Z_{Z\gamma}^{1/2}A^\mu\right] \\
&\quad \times\partial_\mu \left[Z_{GA}^{1/2}A + Z_G^{1/2}G\right]
\end{aligned}$$

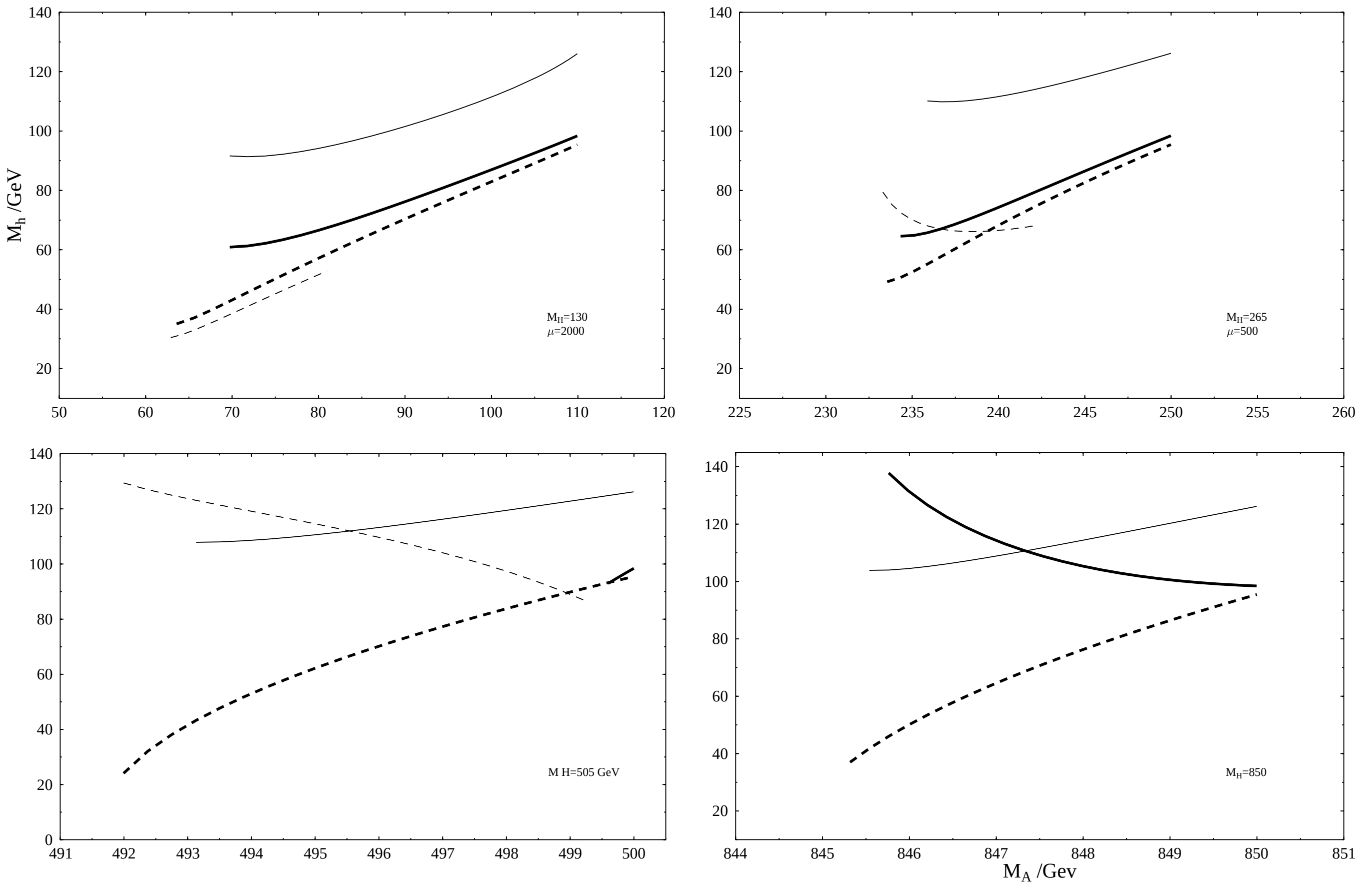

Fig. 1. A radiatively corrected light CP-even Higgs mass is plotted as a function of M_A, $\tan\beta$ varying implicitly from 1.4 (small M_A) to 80 (large M_A), with M_H fixed. The solid (dashed) lines are for the heavy (light) stops, with $\mu = 0$ (deferent μ marked for $\theta_{\tilde{t}} = 0$), and the thin (thick) lines are for zero (maximal) mixing

$$
\begin{aligned}
&= -M_Z \left[\frac{1}{2}\delta Z_Z + \frac{1}{2}\delta Z_G + \frac{\delta M_Z}{M_Z}\right] \quad Z^\mu \partial_\mu G \\
&\quad -M_Z \left[\frac{1}{2}\delta Z_{GA}\right] \quad Z^\mu \partial_\mu A - M_Z \left[\frac{1}{2}\delta Z_{Z\gamma}\right] \quad A^\mu \partial_\mu G \\
&\quad +0 \quad A^\mu \partial_\mu A
\end{aligned} \tag{4.11}
$$

Analogous cases take place for electric charged gauge-scalar mixing and all other truncated Green functions. This means simply that the WTI has ensured that the renormalization *with* $\mathcal{L}_{\rm gf}$ *left out*, can cancel all the UV divergences.

In fact, we could have renormalized $\mathcal{L}_{\rm gf}$ with explicit renormalization constants like the last line in (3.2). As a consequence, we would have to seek proper counterterms for α_z [9], so that all these divergences could be canceled within the $\mathcal{L}_{\rm gf}$ terms. We have not followed such a method, and we prefer to succeed the inference of the work [28] to economize on the renormalization of $\mathcal{L}_{\rm gf}$ at the beginning.

5 Discussions and conclusions

When such a systematic renormalization scheme of Higgs sector and gauge-scalar mixing is completed, the calculation of the S matrix can be organized in an apparent and simple way, like in [13] and [15]. Here we just want to know by which feature and to what extent we can judge whether the lightest Higgs is of SUSY or not, if it is finally excited somewhere. Thus (3.19, 3.23,3.21, and 3.24) have to be employed for a complete simulation. Equation (2.8) sets the mass of the lightest and the charged Higgs when they appear as virtual particles in the inner line of loops. A symbolic $M^2_{hP}(M^2_{H+P})$ can save bookkeeping for the physical mass of the light CP-even (charged) Higgs and the scheme can undoubtedly also work out their magnitudes (the radiative correction to M^2_{hR} or M^2_{H+R}).

After the Taylor expansion in (3.19), the manifestation of the UV cancellation is straightforward in our analysis expressions. For it, we use expressions such as $\cos 2\alpha = -\cos 2\beta(M_A^2 - M_Z^2)/(M_H^2 - M^2_{hR})$, $M^2_{hR} + M_H^2 = M_A^2 + M_Z^2$ and a relation in the stop sector $2M_t(A_u + \mu\cot\beta) = (M^2_{\tilde{t}_1} - M^2_{\tilde{t}_2})\sin 2\theta_{\tilde{t}}$ (where $\theta_{\tilde{t}}$ is the mixing angle between the left and the right hand stop quarks). However, that series is not convenient for a numerical solution, so we iterate q^2 near M^2_{hR} using a standard FF package [29] until our (3.19) is satisfied. In our scheme, the expression (3.20) means that we can not make a *global* plot for M_{hP} against M_A (M_H) when M_H (M_A) is fixed. We investigated the range $M_A(M_H) \sim 110, 250, 500, 850$ GeV respectively with a top quark mass $M_t = 175$ GeV as shown in Fig. 1. As to the parameters in the stop quark sector, we prefer to the physical masses of stops and their mixing angle. We survey both zero ($\theta_{\tilde{t}} = 0$) and maximal ($\theta_{\tilde{t}} = \pi/4$) stop mixing and, moreover, for each value of $\theta_{\tilde{t}}$ we investigated both *light* stop spectra ($M_{\tilde{t}_1} = 70$ $M_{\tilde{t}_2} = 230$, just over experiment bound [30]) and *heavy* spectra ($M_{\tilde{t}_1} = 250$ $M_{\tilde{t}_2} = 850$ GeV, near TeV). We also reproduced the conventional plot in Fig. 2 to show $\tan\beta$ dependence through (2.5).

One can still recognize M_{hP} from the profile in our Fig. 1, although they may seem a little unfamiliar to eyes accustomed to Fig. 2. The $\tan\beta \sim 1$ protrusion near $M_A \sim 350$ GeV in Fig. 2, which is the threshold effect of the virtual particles in the scalar integrals (B_0 functions), can be boxed out if the curves are plotted within a M_A range as small as [8,23]. After the comparison of our Fig. 2 with Fig. 2a in [7], Fig. 1 in [8] with our Fig. 1, and the figures in [23], one can conclude that the dominant radiative corrections for the lightest Higgs has been acquainted properly, although a more accurate prediction for its mass is not reached in this paper, since neither the whole virtual particles nor the two-loop effect were included in our numerical iteration.

It is indeed well known that there exists a UV-finite difference in the different renormalization of a parameter. Nevertheless, the difference for $\delta\beta$ is within the current order but not in the next order even given the same mass-shell scheme. We have systematized the renormalization of fields and parameters into a complete FDC procedure, which is designed for a simple and consistent amplitudes calculation with neither EP nor RG, since less junction means less uncertainty. This realization is also compatible with the taking over of the conventional treatment in [12,9,13,14] for the renormalization of SM gauge bosons, fermions, and couplings. If the Higgs and stop sector data are measured in future experiments, one can make a more meticulous simulation to test which realization of perturbative expanding will approach fast and fit well. Furthermore, given a possible parameter Δ_h (like the Δ_r [31] in SM), (2.5) may hold, even if future experiments give us an alternative observable with more precision than M_H or M_A.

This systematic renormalization for the MSSM necessitates the SUSY partners of the involved virtual particles, no matter how heavy those partners would be. Otherwise the UV divergence would be left in the mass of the light CP-even (charged) Higgs boson or in the physical amplitude in calculations. The decoupling theorem [19,32] still holds, but in a mode in which a particle has to decouple with its SUSY partner when UV divergence is needed to cancel, while a particle will suppress its contribution to the virtual correction when its mass is much larger than the energy scale of the physics. If one uses the decoupling theorem in a manner such that all the fields of SUSY partners are integrated out from the original MSSM Lagrangian, i.e., one follows the scheme for the 2HDM in [14], then the price is that the angles β and α and all the masses of the Higgs bosons have to be input as independent FDC parameters. Anyway, that 2HDM and this MSSM have the same gauge structure, so our (4.9,4.10) are still valid and utilizable.

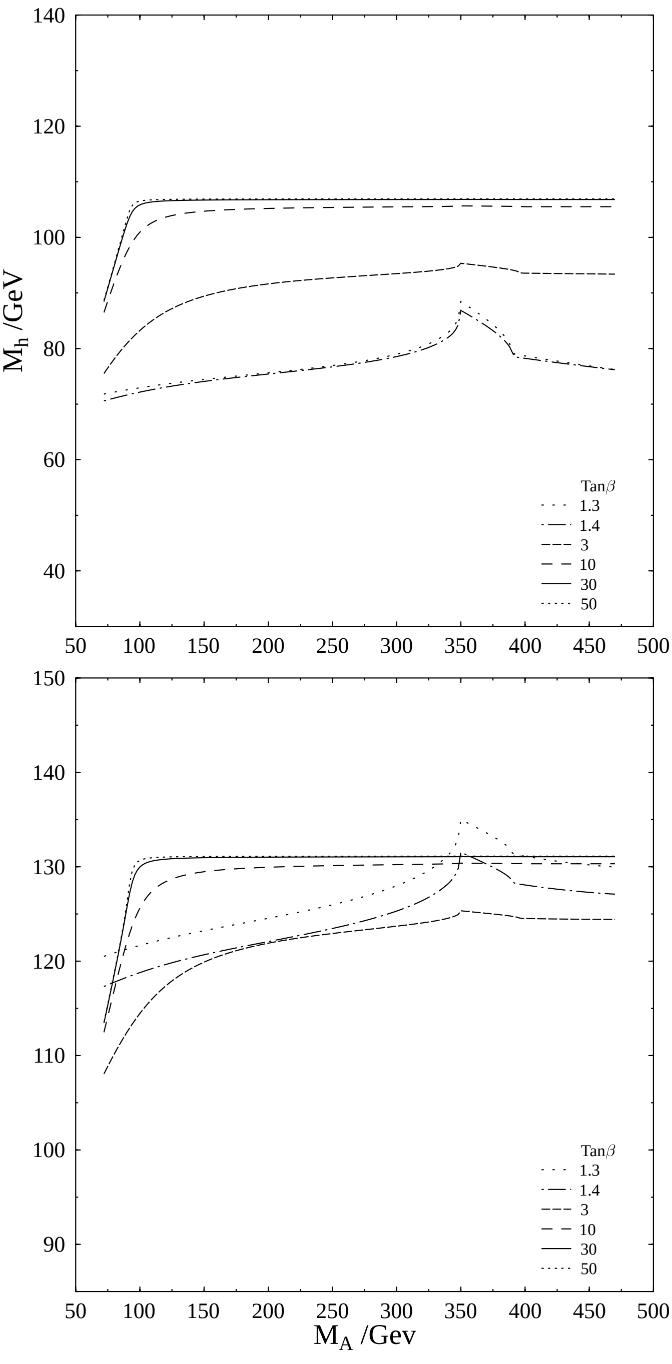

Fig. 2. A radiatively corrected light CP-even Higgs mass is plotted against an effective $\tan\beta$, where a moderate representative magnitude is adapted randomly for other parameters. (For example, $50 \sim M_A \sim 500$ GeV, $M_{\tilde{t}_1} = 200$ $M_{\tilde{t}_2} = 500$ GeV, $\mu = 0$). Stops have zero mixing $\theta_{\tilde{t}} = 0$ in the upper figure and get their maximal $\theta_{\tilde{t}} = \pi/4$ in the lower figure

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A Appendix A: Mass vs. gauge eigenstates in the MSSM

1. MSSM field representation
The MSSM field representation is

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}, \begin{pmatrix} G \\ A \end{pmatrix} = \begin{pmatrix} c_\vartheta & s_\vartheta \\ -s_\vartheta & c_\vartheta \end{pmatrix} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix}, \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} c_{\vartheta^+} & s_{\vartheta^+} \\ -s_{\vartheta^+} & c_{\vartheta^+} \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}, \tag{A.1}$$

where $c_{\vartheta^+} = \cos\vartheta^+,\ s_{\vartheta^+} = \sin\vartheta^+$ and so on. At tree level, $\vartheta = \vartheta^+ = \beta$. This renormalization can accommodate even the renormalized $\vartheta^R = \vartheta^R_+ = \beta^R$ and $\delta\vartheta = \delta\vartheta^+ = \delta\beta$.

2. m_i^2 expressed as physical parameters
One can solve $m_1^2,\ m_2^2,\ m_3^2$ as a function of $\mathcal{M}_{AA},\ \mathcal{T}_H,\ \mathcal{T}_h$ in the MSSM:

$$\begin{aligned} m_1^2 = & -\sec^2(\beta-\vartheta)\{[-16\mathcal{M}_{AA} + 2(8\mathcal{M}_{AA} + 2g^2v^2 \\ & \times \cos^2(\beta-\vartheta))\cos 2\beta]/32 - \mathcal{T}_H\cos\vartheta[\cos(\alpha-\beta-\vartheta) \\ & -2\cos(\alpha+\beta-\vartheta) - \cos(\alpha-\beta+\vartheta)]/(2v) \\ & -\mathcal{T}_h\cos\vartheta[-\sin(\alpha-\beta-\vartheta) + 2\sin(\alpha+\beta-\vartheta) \\ & +\sin(\alpha-\beta+\vartheta)]/(2v)\} \\ m_2^2 = & \sec^2(\beta-\vartheta)\{[16\mathcal{M}_{AA} + 2(8\mathcal{M}_{AA} + 2g^2v^2 \\ & \times\cos^2(\beta-\vartheta))\cos 2\beta]/32 + \mathcal{T}_h\sin\vartheta[\cos(\alpha-\beta-\vartheta) \\ & +2\cos(\alpha+\beta-\vartheta) + \cos(\alpha-\beta+\vartheta)]/(2v) \\ & +\mathcal{T}_H\sin\vartheta[\sin(\alpha-\beta-\vartheta) + 2\sin(\alpha+\beta-\vartheta) \\ & +\sin(\alpha-\beta+\vartheta)]/(2v)\} \\ m_3^2 = & \sec^2(\beta-\vartheta)\{\mathcal{T}_h[\cos(\alpha+\beta) + \cos(\alpha-\beta)\cos 2\vartheta]/(2v) \\ & +\mathcal{T}_H[\cos 2\vartheta\sin(\alpha-\beta) + \sin(\alpha+\beta)]/(2v) \\ & -\mathcal{M}_A^2\sin 2\beta/2\}. \end{aligned} \tag{A.2}$$

3. Mass form
Multiplied by a factor of 2, the matrix elements rotated from the original potential read

$$\begin{aligned} \mathcal{M}_{AG} = \mathcal{M}_{GA} = & [8m_3^2\cos 2\vartheta + (-4m_1^2 + 4m_2^2 - g_1^2v_1^2 \\ & -g_2^2v_1^2 + g_1^2v_2^2 + g_2^2v_2^2)\sin 2\vartheta]/4 \\ \mathcal{M}_{GG} = & [(4m_1^2 - 4m_2^2 + g_1^2v_1^2 + g_2^2v_1^2 - g_1^2v_2^2 - g_2^2v_2^2) \\ & \times\cos 2\vartheta + 4(m_1^2 + m_2^2 + 2m_3^2\sin 2\vartheta)]/8 \\ \mathcal{M}_{hh} = & [4m_1^2 + 4m_2^2 + g_1^2v_1^2 + g_2^2v_1^2 + g_1^2v_2^2 + g_2^2v_2^2 \\ & -2(2m_1^2 - 2m_2^2 + g_1^2v_1^2 + g_2^2v_1^2 - g_1^2v_2^2 - g_2^2v_2^2) \\ & \times\cos 2\alpha + 2(-4m_3^2 + g_1^2v_1v_2 + g_2^2v_1v_2) \\ & \times\sin 2\alpha]/8 \\ \mathcal{M}_{hH} = \mathcal{M}_{Hh} = & [(4m_3^2 - (g_1^2+g_2^2)v_1v_2)\cos 2\alpha \\ & +(-2m_1^2 + 2m_2^2 - g_1^2v_1^2 - g_2^2v_1^2 + g_1^2v_2^2 + g_2^2v_2^2) \\ & \times\sin 2\alpha]/2 \\ \mathcal{M}_{H^+H^-} = & (4m_1^2 + 4m_2^2 + g_2^2v_1^2 + g_2^2v_2^2 \\ & +(-4m_1^2 + 4m_2^2 - g_1^2v_1^2 + g_1^2v_2^2)\cos 2\vartheta^+ \\ & +2(-4m_3^2 + g_2^2v_1v_2)\sin 2\vartheta^+]/4 \\ \mathcal{M}_{H^+G^-} = \mathcal{M}_{G^+H^-} = & [2(4m_3^2 - g_2^2v_1v_2)\cos 2\vartheta^+ \\ & +(-4m_1^2 + 4m_2^2 - g_1^2v_1^2 + g_1^2v_2^2)\sin 2\vartheta^+]/4 \\ \mathcal{M}_{G^+G^-} = & (4m_1^2 + 4m_2^2 + g_2^2v_1^2 + g_2^2v_2^2 \\ & +(4m_1^2 - 4m_2^2 + g_1^2v_1^2 - g_1^2v_2^2)\cos 2\vartheta^+ \\ & +2(4m_3^2 - g_2^2v_1v_2)\sin 2\vartheta^+]/4. \end{aligned} \tag{A.3}$$

B Appendix B: Counterterms in the MSSM

1. The counterterms for the combinations of tadpoles are

$$\begin{aligned} T_{AA} &= 0 \\ T_{HH} &= \frac{-1}{2v}\cos(\alpha-\beta)[-3T_H + T_H\cos 2(\alpha-\beta) \\ & \quad -T_h\sin 2(\alpha-\beta)] \\ T_{AG} &= T_{GA} = \frac{2}{v}[T_h\cos(\alpha-\beta) + T_H\sin(\alpha-\beta)] \\ T_{GG} &= \frac{1}{v}[T_H\cos(\alpha-\beta) - T_h\sin(\alpha-\beta)] \\ T_{hh} &= \frac{-1}{2v}\ \sin(\alpha-\beta)[3T_h + T_h\cos 2(\alpha-\beta) \\ & \quad +T_H\sin 2(\alpha-\beta)] \\ T_{Hh} &= T_{hH} = \frac{1}{2v}[3T_h\cos(\alpha-\beta) + T_h\cos 3(\alpha-\beta) \\ & \quad -4T_H\sin^3(\alpha-\beta)] \\ T_{H^+H^-} &= 0 \\ T_{H^+G^-} &= T_{G^+H^-} = \frac{2}{v}[T_h\cos(\alpha-\beta) + T_H\sin(\alpha-\beta)] \\ T_{G^+G^-} &= \frac{2}{v}[T_H\cos(\alpha-\beta) - T_h\sin(\alpha-\beta)]. \end{aligned} \tag{B.1}$$

These tadpole corrections in CP-even neutral Higgs are different from the ones obtained by others [15,14]. It is easy to check that our $T_{HH} + T_{hh}$ is equal to the $b_{HH} + b_{hh} - b_{AA}$ in [15], and the a_{GG} in [19]. However, the Goldstone-Higgs mixing terms are the same, as pointed in the context, these Goldstone-Higgs tadpole loops must be included in the corresponding proper vertex.

2. Renormalization for charged Higgs sector
Pole mass and H^+G^+ mixing to one-loop order can be written as

$$\begin{aligned} & \Sigma_{H^+}(q^2) + (q^2 - M^2_{H^+R})Z_{H^+} - \delta M^2_{H^+} = 0 \\ & \delta M^2_{H^+} = \delta M_A^2 + \delta M_Z^2 \\ & \Sigma_{H^+G^+}(0) + (0 - M^2_{H^+R} \\ & \quad -\delta M^2_{H^+})Z^{1/2}_{H^+}Z^{1/2}_{H^+G^+} - T_{G^+H^+} = 0 \\ & \Sigma_{H^+G^+}(q^2) + (q^2 - M^2_{H^+R} - \delta M^2_{H^+})Z^{1/2}_{H^+}Z^{1/2}_{H^+G^+} \\ & \quad +Z^{1/2}_{G^+G^+}Z^{1/2}_{G^+H^+}q^2 - T_{G^+H^+} = 0 \\ & \frac{\mathrm{d}}{\mathrm{d}q^2}\Sigma_{H^+}(q^2) + Z_{H^+} + Z_{H^+G^+} = 1. \end{aligned} \tag{B.2}$$

The third equation restricts the charged Goldstone pole mass to be zero, and it is helpful to solve $\delta Z_{G^+H^+}$ in the fourth equation, which is more useful for physical processes. There is an analogous expression for the neutral Goldstone, but the δZ_{GA} can be calculated independently.

C Appendix C: Some WTI in the neutral sector of the MSSM

The following Ward–Takahashi identities hold in the neutral sector of the MSSM:

$$\begin{aligned}
&G^{\mu\nu}_{ZZ}\tilde{\Gamma}[C^Z, K^\nu_Z] + G^{\mu\nu}_{Z\gamma}\tilde{\Gamma}[C^Z, K^\nu_\gamma] \\
&\quad + G^{\mu}_{ZG}\tilde{\Gamma}[C^Z, K_G] + G^{\mu}_{ZA}\tilde{\Gamma}[C^Z, K_A] = 0 \\
&G^{\nu}_{GZ}\tilde{\Gamma}[C^Z, K^\nu_Z] + G^{\nu}_{G\gamma}\tilde{\Gamma}[C^Z, K^\nu_\gamma] \\
&\quad + G_{GG}\tilde{\Gamma}[C^Z, K_G] + G_{GA}\tilde{\Gamma}[C^Z, K_A] = 0 \\
&G^{\nu}_{AZ}\tilde{\Gamma}[C^Z, K^\nu_Z] + G^{\nu}_{A\gamma}\tilde{\Gamma}[C^Z, K^\nu_\gamma] \\
&\quad + G_{AG}\tilde{\Gamma}[C^Z, K_G] + G_{AA}\tilde{\Gamma}[C^Z, K_A] = 0 \\
&G^{\mu\nu}_{\gamma Z}\tilde{\Gamma}[C^Z, K^\nu_Z] + G^{\mu\nu}_{\gamma\gamma}\tilde{\Gamma}[C^Z, K^\nu_\gamma] \\
&\quad + G^{\mu}_{\gamma G}\tilde{\Gamma}[C^Z, K_G] + G^{\mu}_{\gamma A}\tilde{\Gamma}[C^Z, K_A] = 0 \\
&G^{\mu\nu}_{\gamma Z}\tilde{\Gamma}[C^\gamma, K^\nu_Z] + G^{\mu\nu}_{\gamma\gamma}\tilde{\Gamma}[C^\gamma, K^\nu_\gamma] \\
&\quad + G^{\mu}_{\gamma G}\tilde{\Gamma}[C^\gamma, K_G] + G^{\mu}_{\gamma A}\tilde{\Gamma}[C^\gamma, K_A] = 0 \\
&G^{\nu}_{AZ}\tilde{\Gamma}[C^\gamma, K^\nu_Z] + G^{\nu}_{A\gamma}\tilde{\Gamma}[C^\gamma, K^\nu_\gamma] \\
&\quad + G_{AA}\tilde{\Gamma}[C^\gamma, K_G] + G_{AA}\tilde{\Gamma}[C^\gamma, K_A] = 0. \qquad (C.1)
\end{aligned}$$

Similar expressions hold for the charged sector and lead to the W mixing with scalars,

$$\begin{aligned}
\delta Z_{W^+G^+} &= \frac{1}{2}\delta Z_W + \frac{1}{2}\delta Z_{G^+} + \frac{\delta M_W^2}{2M_W^2} \\
\delta Z_{W^+H^+} &= \frac{1}{2}\delta Z_{G^+H^+}. \qquad (C.2)
\end{aligned}$$

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